Option Pricing as a Proxy for Discount for Lack of Marketability in Private Company Valuations

A Working Paper

by DAVID B. H. CHAFFE III

The effect of marketability on value has been widely reviewed in financial articles. One aspect of the issue is clear; that pricing improves with greater marketability, all other factors being equal. The difficulty lies in quantifying the extent of price difference or discount as marketability is impaired or ceases to be present. The principal economic factor causing the discount is the increase in risk caused by the inability to quickly and efficiently return the investment to a cash position. There is also a component of the discount that is related to the inability to realize an intermediate gain quickly and efficiently. For purposes of this analysis, we forego quantification of the discount factor associated with this second aspect of marketability.

The marketability issue is present in the sale of restricted stock by a company with publicly traded shares, in which case a holding period of two years is required before resale is permitted under the registration exemption provisions of Rule 144 of the Securities and Exchange Commission. It is evidence taken from several studies of the restricted/traded share comparison that is most often cited as a proxy for the discount for lack of marketability in private company valuation.

When provided with an option to sell, otherwise non-marketable shares are given marketability. (For instance, we see this type of provision in Employee Share Ownership Plans where, in such cases, marketable level values are found).

Following this logic, the cost or price of the option to sell (a put option) represents all (or a major portion) of the discount to be taken from the marketable price to price the non-marketable shares.

To summarize, if one holds restricted or non-marketable stock and purchases an option to sell those shares at the free market price, the holder has, in effect, purchased marketability for the shares. The price of the put is the discount for lack of marketability.

With some adjustment for simplification, we can examine this theory as it relates to valuation of private company shares.

- As an option pricing model use the European option which is exercisable only at the end of the option period. This gives us a model that is quite similar to the Rule 144 holding period in restricted shares.
- The option price (strike price) is the freely traded (marketable) price at time of purchase (valuation date).

These choices are consistent with the nature of private company shares, noting that with a private company there may never be a public market for the shares. As in all marketability studies to date, however, it is necessary to synthesize or assume a marketable relationship (as in the lettered stock studies and pre and post public market studies, etc.). Thus our models assume marketability of the shares at the end of the holding period which is the same date as

the expiration (exercise) date of the option. The holder of the paired issues (option and stock) cannot have marketability until expiration of the holding (option) period. The holder then has the market price of the shares or the marketable price of the shares at date of purchase, whichever is greater.

The use of the European option model will result in lower option prices than if the American option form is used or if variations are considered to give the effect of a perpetual put adjusted constantly to the then marketable price. The findings will therefore err to less discount or the minimum applicable discount.

Therefore, if we consider the money difference between marketable and non-marketable securities to be the cost of an option to sell (strike at the marketable price, and for a term equal to the time the securities are expected to remain non-marketable), the application of the put pricing formula may provide an approach to defining the discount for lack of marketability.

The Black-Scholes simplified option pricing model defines the following variables:

1. Stock Price (market price);
2. Strike Price (option price);
3. Time to Expiration;
4. Interest Rate (cost of capital); and,
5. Volatility.

Of these items, Volatility is the most difficult to define in the context of a private company. This variable is normally calculated using historical price fluctuations in traded markets. When valuing privately held securities, estimates can be made by selecting an appropriate public market proxy, then using the volatility for the proxy in place of a specific calculation.

The Black-Scholes Put formula is as follows:

$$P = Kr^{-t}N(y+\sigma\sqrt{t}) - SN(y)$$

where:

$$y = \frac{\log (Kr^{-t}/S)}{\sigma\sqrt{t}} - \tfrac{1}{2}\,\sigma\sqrt{t}$$

P = put option price
S = stock price
K = strike price
r = interest rate (+1.0 in yield)
t = time to expiration (% of year)
σ = volatility
$N(z)$ = standard normal (distribution) density function

$$N(z) \approx 1 - (1/\sqrt{2\pi})\, e^{-z^2/2}\, (b_1k+b_2k^2+b_3k^3+b_4k^4+b_5k^5)$$

$k \equiv 1/(1+az)$
$a = 0.2316419$
$b_1 = 0.319381530$
$b_2 = 0.356563782$
$b_3 = 1.781477937$
$b_4 = -1.821255978$
$b_5 = 1.330274429$

The formula produces prices that vary directly with time and volatility and inversely with interest rate. (The time value of the option price begins to erode after approximately 6.6 years producing a slight down-curve in prices in longer options). It is not surprising that the variables affecting the put pricing model are quite similar to those noted in the Revenue Ruling 77-287 discussion of factors affecting discounts in unregistered shares.

In the context of the pricing formula, volatility is an expression of the total risk of a stock and is most frequently derived using historical price fluctuations of the stock in question. Volatility for small companies traded in the over the counter (OTC) market would, in most cases, be greater than those of NASDAQ traded issues or national exchange listed securities. The volatility of privately held shares can only be estimated by comparing the private company's financial and operating data with those of comparative public companies for which volatility can be determined. We note that volatility is low for large capitalization, actively traded issues and trends sharply upward for shares of smaller companies or highly speculative companies. This trend implies that volatility of shares of a small, privately held company might be at least 60%. As a point of reference, Exhibit III is a list of certain publicly traded common stocks and common stock indices with their respective volatility factors in April 1993. The list was selected only to give examples over a range of volatility figures. The volatility given in this table is a market implied volatility calculated empirically from the then traded put option price.

To model put prices over a range of volatilities, we make the following assumptions:

1. No costs are involved in marketing the security.
2. No dividends or distributions are paid on the security.
3. Interest rates (cost of capital) are constant in the future.
4. Sale of the security [exercise of the option] cannot occur before the last day of the option term (European Option).

Applying the Black-Scholes put formula over a range of time, and varying volatility, we see results as shown in Exhibit I. As the chart at Exhibit I shows, the put price rises rapidly to approximately year two, with the slope of the curves flattening as time increases thereafter. (Interest rate is fixed at 5.0% for purposes of calculating the curves shown in Exhibit I).

Reviewing Exhibit I, considering that volatility for shares of most smaller, privately held companies fit the "VOL 60%-70%-80%-90%" curves, a range of put prices of approximately 28% to 41% of the marketable price is shown at the two-year intercept. At the four-year intercept, these ranges are 32% to 49%, after which time increases do not substantially change the put price.

These put prices, expressed as percentages of the market price (strike price), offer us an estimate of the discount for lack of marketability. The discount ranges of 28% to 49% are quite similar to those developed using data from the market evidence studies of marketability such as the SEC Institutional Investor Study, the Robert E. Moroney studies and the John D. Emory data.

Sensitivity to interest rate assumptions (cost of capital) can be noted from Exhibit II which varies interest rate, holding other factors constant. This chart shows declining put prices as interest rates increase. The change from the 5% intercept to the 9% intercept is approximately five percentage points. There is some market evidence (subject to further research by the writer) that put prices in the traded options market do not decline during periods of high interest rates, thereby indicating higher implied volatility factors that offset the purely mathematical calculation varying interest rates, holding all other factors constant.

Investors may not actively and consciously apply option pricing in determining discounts

in actual transactions involving restricted shares of public companies. It is the writer's view that the put is a proxy for marketability (it certainly is in ESOP situations) and the option pricing formula may yield a valid confirmation of the market-based models that collect and analyze transaction data.

In releasing this article, I am aware that there are many theoretical and practical limits to the use of any formula in financial market determinations. However, the discount relationships that I have noted earlier seem more than coincidental to me. I welcome your comments and suggestions for further study.

Endnotes

1. There is also a capacity to repurchase issue that, for discussion purposes, we assume is adequate.
2. The European option contrasts with the American option which permits exercise at any time that the option is outstanding.

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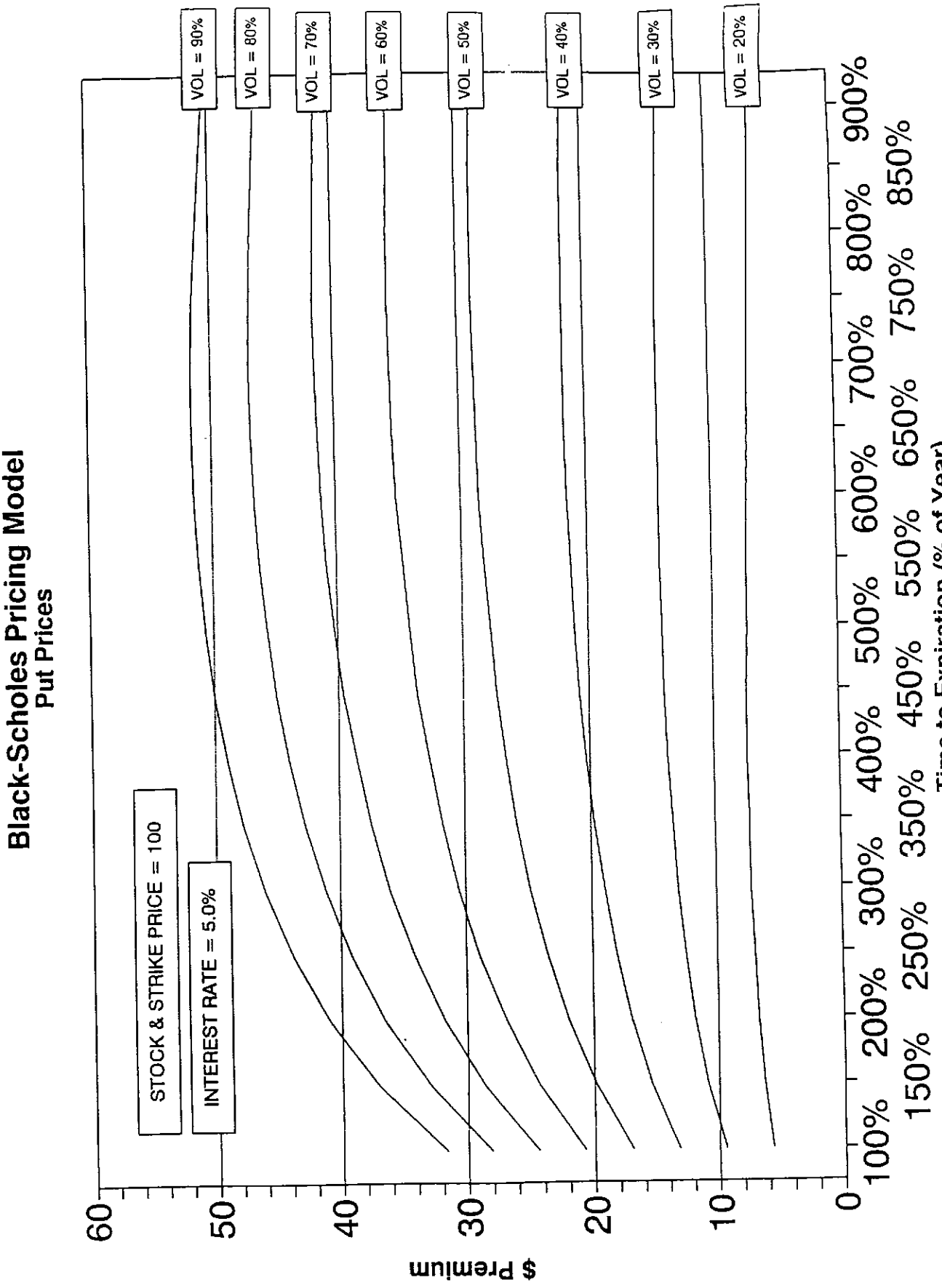
Black-Scholes Pricing Model
Put Prices
STOCK & STRIKE PRICE = 100
INTEREST RATE = 5.0%
VOL = 90%
VOL = 80%
VOL = 70%
VOL = 60%
VOL = 50%
VOL = 40%
VOL = 30%
VOL = 20%
$ Premium
0
10
20
30
40
50
60
100% 150% 200% 250% 300% 350% 400% 450% 500% 550% 600% 650% 700% 750% 800% 850% 900%
Time to Expiration (% of Year)

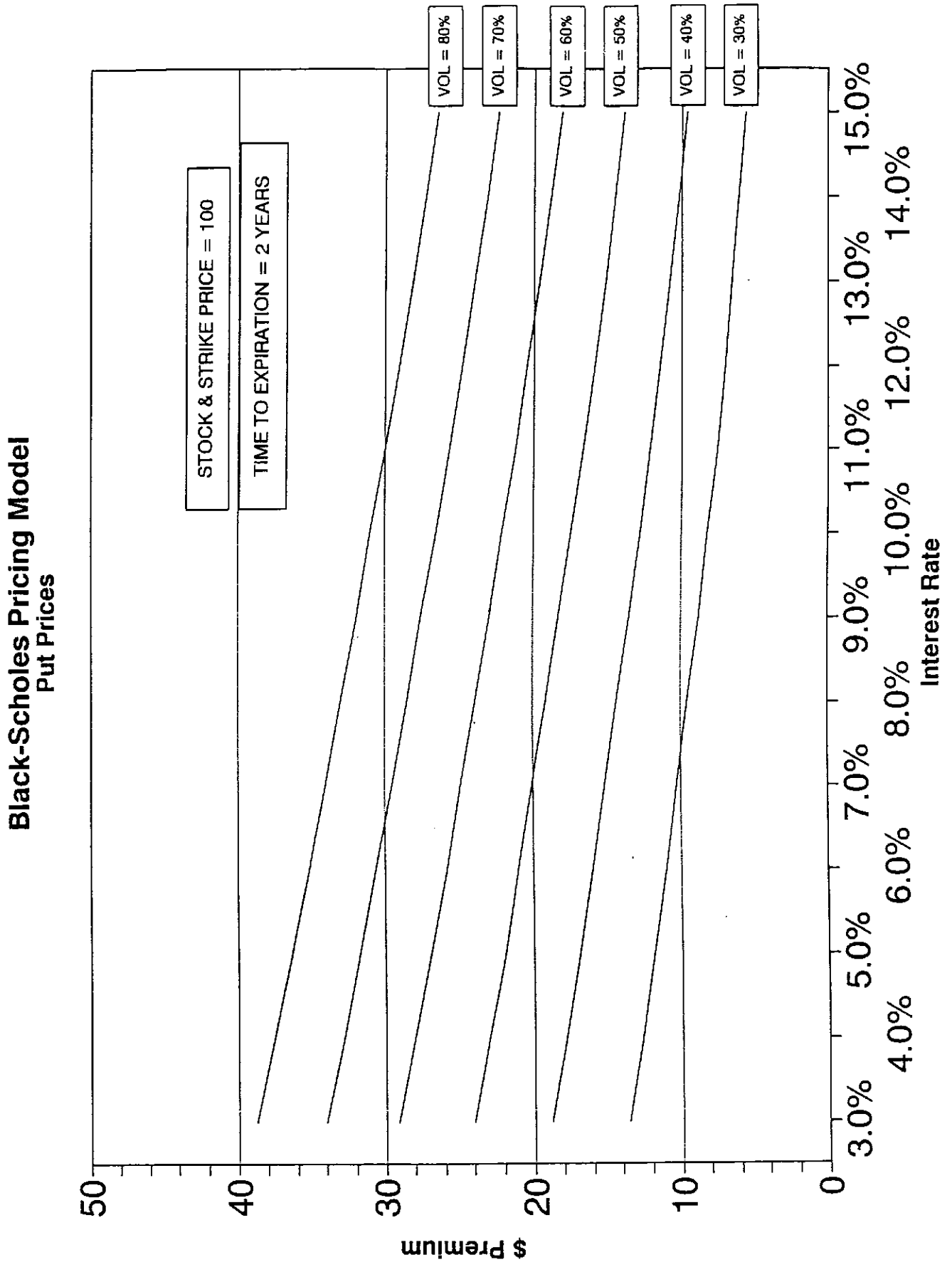
Black-Scholes Pricing Model
Put Prices
STOCK & STRIKE PRICE = 100
TIME TO EXPIRATION = 2 YEARS
VOL = 80%
VOL = 70%
VOL = 60%
VOL = 50%
VOL = 40%
VOL = 30%
$ Premium
0
10
20
30
40
50
3.0%
4.0%
5.0%
6.0%
7.0%
8.0%
9.0%
10.0%
11.0%
12.0%
13.0%
14.0%
15.0%
Interest Rate

EXHIBIT III

LIST OF INDEXES COMPANIES WITH TRADED OPTIONS AND RELATED MARKET HISTORICAL VOLATILITY (expressed as decimal)

April 1993

Name	Volatility
S&P 500	.138
S&P 100	.133
Value Line Index	.083
Russell 2000 Index	.111
Wilshire Small Cap Index	.135
Amex Oil & Gas Index	.168

		Stock Price	Exchange*
Alcan Aluminum	.491	18.50	NYSE
American Brands	.485	30.00	NYSE
Becton-Dickinson	.277	34.875	NYSE
Boeing Company	.279	34.875	NYSE
Coca Cola Corp.	.413	41.00	NYSE
Commonwealth Edison	.312	27.875	NYSE
Digital Equipment	.507	42.75	NYSE
Exxon	.200	67.00	NYSE
General Electric	.193	89.50	NYSE
GTE	.240	37.00	NYSE
IBM	.389	52.625	NYSE
Johnson & Johnson	.310	41.375	NYSE
Smith-Kline Beecham	.498	27.25	NYSE
Xerox Corp.	.270	80.875	NYSE
Affymax A.G.	.949	15.75	NMS
Airborne Freight	.530	23.00	
Amtech Corp.	.811	32.75	NMS
Ask Computer Systems	.834	11.375	NMS
Brunos, Inc.	.896	8.875	NMS
Clothestime, Inc.	.840	12.75	NMS
G Tech Holdings	.712	35.50	NYSE
Medtronic, Inc.	.669	66.25	NYSE
National Medical Enterprises	.902	8.625	NYSE
Plains Resources	.829	9.00	ASE
Ryder Systems	.665	27.125	NYSE
SciMed Life Systems, Inc.	.633	38.25	NMS
Showboat, Inc.	.638	21.50	NYSE
Solectron Corp.	.559	40.75	NYSE
Standard Microsystems	.798	16.375	NMS
Synergen, Inc.	.940	10.375	NMS
Tandy Corp.	.587	29.25	NYSE
UJB Financial Corp.	.883	28.125	NYSE
Von's Companies	.548	22.625	NYSE

*NYSE — New York Stock Exchange
ASE — American Stock Exchange
NMS — Nasdaq National Market System